WICHITA STATE UNIVERSITY

Department of Mathematics, Statistics & Physics

The Lecture Series in the Mathematical Sciences Presents Our Guest:

Dr. Christopher Green

Macquarie University, Sydney, Australia

“From bubbles to harmonic measure: special function theory in domains with multiple boundary components”

Abstract:

I will talk about two different problems where a special type of function theory can be used to construct solutions.

The first problem involves solving a type of free boundary problem for bubbles in a Hele-Shaw cell. The addition of surface tension to the bubble boundaries is particularly interesting and plays an important role when finding solutions. With zero surface tension, there is a continuum of bubble speeds for which there are solutions. However, for non-zero surface tension, there is no longer a continuum but instead a discrete, countably infinite family of solutions which exist for each fixed value of the surface tension, with the bubble shapes becoming more exotic as the solution branch number increases. Such problems are dubbed selection problems.

The second problem involves a generalization of harmonic measure. Consider releasing a Brownian particle from a basepoint z_0 in a planar domain $\Omega \subset C$. What is the chance, denoted $h_{\Omega,z_0}(r)$, that the particle's first exit from Ω occurs within a fixed distance $r > 0$ of z_0? The function of r suggested by this question, denoted $h_{\Omega,z_0} :[0,\infty)\to[0,1]$, is called the harmonic measure distribution function, or h-function, of Ω with respect to z_0. We can think of the h-function as a signature that encodes the geometry of the boundary of Ω. In the language of PDEs, the h-function can also be formulated in terms of a suitable Dirichlet problem on Ω. For simply connected domains, the theory of h-functions is now quite well developed and several explicit results are known. However, until recently, for multiply connected domains the theory of h-functions has been almost entirely out of reach.

The special function theory employed when solving the two aforementioned problems is centered around the so-called Schottky-Klein prime function, a special transcendental function which plays a central role in problems involving multiply connected domains (i.e. domains with multiple boundary components). Despite this, it has been scarcely used - until relatively recently - by both pure and applied mathematicians since it was originally written down (independently by both Schottky and Klein towards the end of the 19th century).

Friday, September 21, 2018

3:00 PM in 372 Jabara Hall

Please come join us for refreshments before the lecture at 2:30 p.m. in room 353 Jabara Hall.