WICHITA STATE UNIVERSITY

Department of Mathematics, Statistics & Physics

Visiting Candidate for Assistant Professor Position

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"Aspects of potential theory in the plane, on the sphere, and on the torus"

Abstract:

Consider releasing a Brownian particle from a basepoint z_0 in a planar domain $\Omega \subset \mathbb{C}$. What is the chance, denoted $h_{\Omega,z0}(r)$, that the particle's first exit from Ω occurs within a fixed distance $r > 0$ of z_0? The function $h_{\Omega,z0} : [0, \infty) \to [0, 1]$ is called the harmonic measure distribution function, or h-function, of Ω with respect to z_0. It can also be formulated in terms of a Dirichlet problem on Ω with suitable boundary values. For simply connected domains Ω, the theory of h-functions is now quite well-developed, and in particular the h-function can often be explicitly computed, making use of the Riemann mapping theorem. However, until now, for multiply connected domains the theory of h-functions has been almost entirely out of reach. In this talk, I will show how to construct explicit formulae for h-functions of symmetric multiply connected slit domains whose boundaries consist of an even number of colinear slits. We employ the special function theory of the Schottky–Klein prime function and its associated constructive methods in conformal mapping to build explicit formulae for the h-functions of domains Ω with any finite even number of slits. This is the first time that the h-function of any multiply connected domain has been computed explicitly. I will then show how to generalize these formulae for multiply connected slit domains on the sphere (a genus-0 compact Riemann surface).

I will then talk about the construction of a type of Green's function on a ring toroidal surface (a genus-1 compact Riemann surface). Function theory on the sphere is well-understood and largely motivated by the desire to study various phenomena on Earth. A common approach used to deal with such problems is the stereographic projection of the sphere onto the plane. A natural question is how to extend such investigations to other compact Riemann surfaces. The first closed-form expression for the Green's function of the Laplace-Beltrami operator on a toroidal surface has now been constructed, and I will provide an overview of the mathematical techniques used in its construction. Our technique relies upon the stereographic projection of the torus to a concentric annular plane, wherein the special function theory of the Schottky-Klein prime function may be used. Such a Green's function can also be regarded as a streamfunction for an ideal vortex flow on the surface of the torus.

Monday, March 25, 2019
3:00 PM in 128 Jabara Hall

Please come join us for refreshments before the lecture at 2:30 p.m. in room 353 Jabara Hall.