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Is There an Optimal Size of Fiscal Reserves for Local Governments?

Introduction

As other chapters in this volume have shown, there is wide variation in implementation of fiscal reserves, the levels and types of fiscal reserve funds maintained, and the usage of fiscal reserves during times of financial uncertainty. However, the rationale for maintaining fiscal reserves is widely accepted. As several papers have put it, state and local governments facing a shortfall in revenues available to service desired expenditures have one of four choices:

1. Increase revenues through rate increases or base broadening;
2. Reduce expenditures by eliminating or downsizing programs, restricting eligibility requirements, postponing capital outlays, and deferring maintenance programs;
3. Borrowing money directly from the public through debt issuance or indirectly from fiduciary trust funds; or
4. Use fiscal reserves (see for example Vasche and Williams, 1987).

Of these options, number (1) tends to be tremendously politically unpopular (though some cities accomplished this during the last major economic downturn) and timing of increases can be problematic (it may take time to raise rates or broaden bases, leading to revenue increases just as a recovery is taking place), (2) is unpopular at least in communities who are disproportionately affected by cuts and may produce cuts in programs that are successful in the long-run but simply costly in the short-term, and (3) may be restricted in time (most states are prohibited from using long-term borrowing to fund short-term budget shortfalls) and the time when funds are needed may be exactly the time that it is

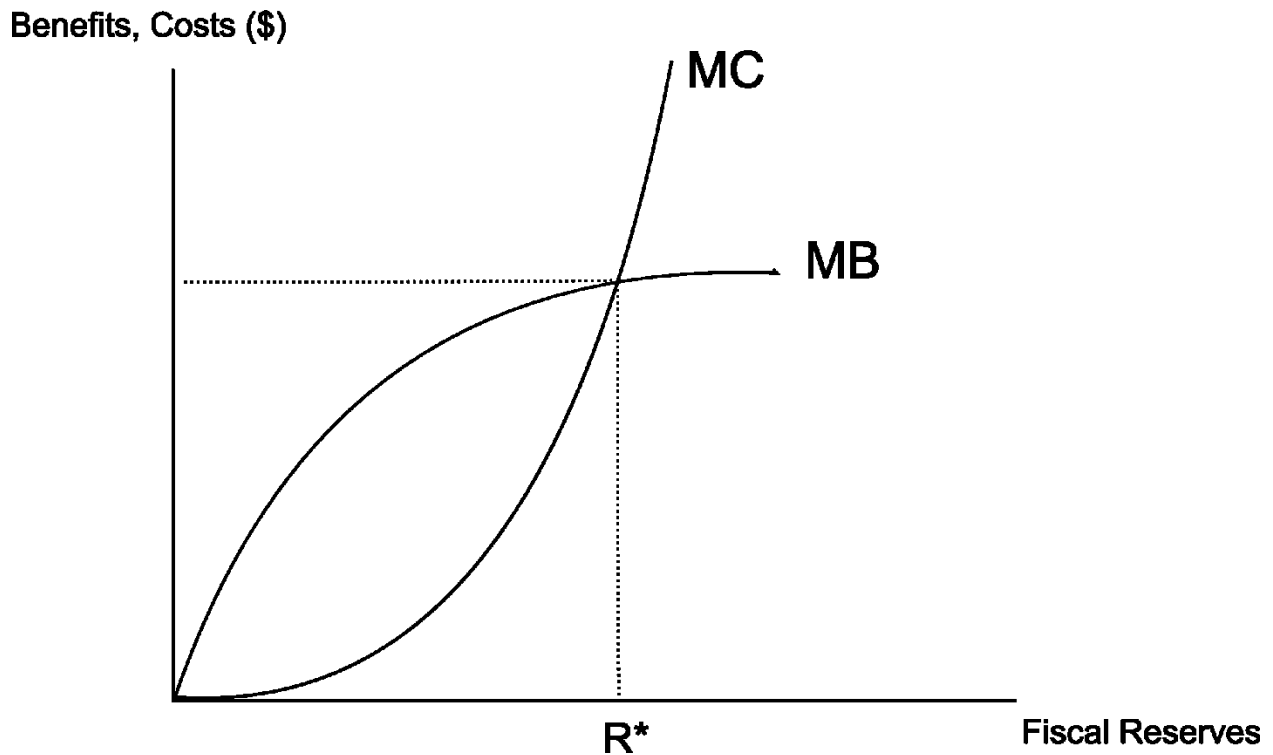
most difficult to borrow. This leaves fiscal reserves as being the most viable (and sometimes only viable) option open to state and local government officials.

There are thus obvious benefits to using fiscal reserves to smooth revenues over the economic cycle.

Fiscal reserves act as a cushion for governments. However, there are costs associated with the accumulation of fiscal reserves. Some are the “opportunity costs” of projects/programs foregone in order to accumulate reserves. Others are political costs of the perceived opportunity for improper use of resources. One can view the decision about the optimal level of reserves as a balancing of those benefits and costs. Figure 1 shows this decision. The marginal benefit of an additional dollar of fiscal reserves is shown as a concave function. At low levels of reserves, the extra benefit of holding an additional dollar of reserves is high. But as the reserves grow, the marginal benefit falls, at some point becoming zero.

The marginal cost of adding a dollar of reserves is initially small, but it grows strongly as more revenue is saved versus put to use providing programs. At point R^* , the marginal benefit of the last dollar of reserves accumulated is exactly equal to its marginal cost, signally the optimal level of reserves has been reached.

Figure 1. The Optimal Level of Reserve Funds.



If that were all there was to be said about the analysis, determining the optimal level of reserves would be an easy task. However, calculating the marginal benefit of a given level of reserves is extraordinarily difficult due to some fundamental challenges. These include:

1. A need to decide on the appropriate measure of reserves;
2. The analysis of necessary levels of reserves involves stochastic uncertainty;
3. The asymmetric nature of uncertainties;
4. The shifting nature of public finances;
5. The need for an implementation plan that recognizes a true need for the use of reserves versus a “manufactured need”.

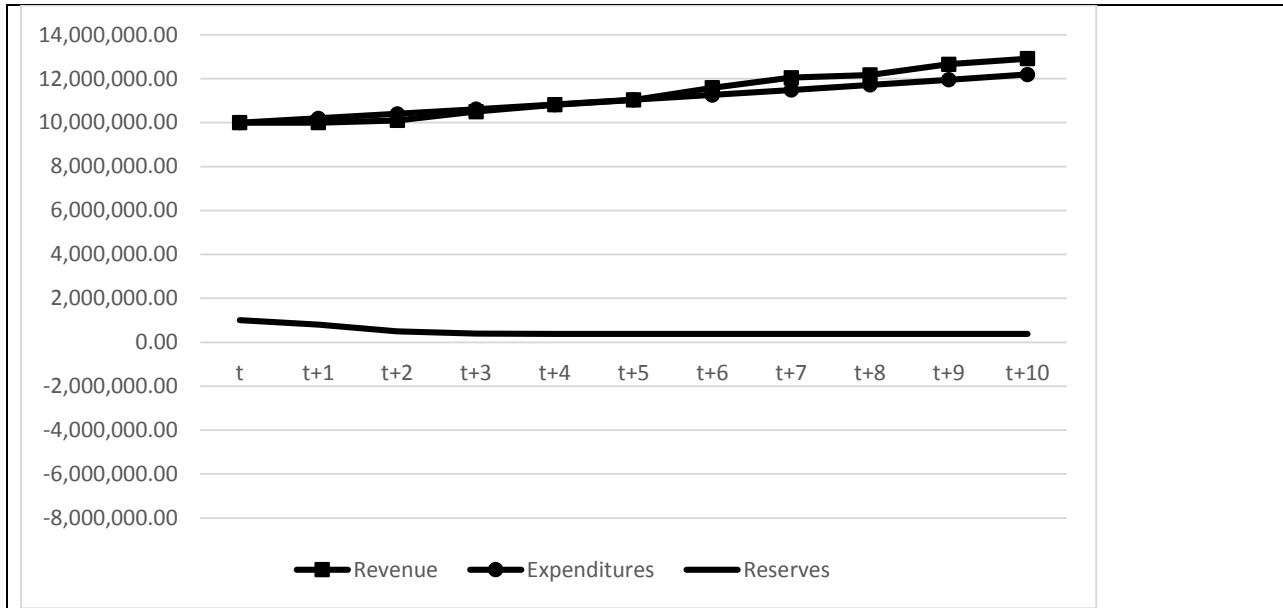
We next consider these in turn. With respect to the appropriate measure of reserves, theoretically governments have many potential sources of reserves. There are budget stabilization funds, sometimes called “rainy day funds”, where governments can formally set aside revenues in a savings account for future use. There are also unreserved fund balances in the General Fund and other Governmental Funds which theoretically are available for spending in future years. Finally, there may be unrestricted balances that have accrued in other funds (including in Fiduciary Funds such as trust funds) which can be spent. This last category is the most contentious, although many state and local governments made transfers from many funds into their General Funds during the last economic downturn in order to stabilize finances (Williams, 2012).

Stochastic uncertainty affects the determination of optimal levels of fiscal reserves directly. The need for fiscal reserves plays out many years if not decades into the future. As with revenue forecasting, errors in determining needs for expenditure and realizations of revenues (which are fundamental to the calculation of optimal budget reserves) are likely. The size of future forecasting errors can be estimated based on past data, but if a fundamental “structural break” happens in the finances of an organization, future errors may be much larger than past errors.

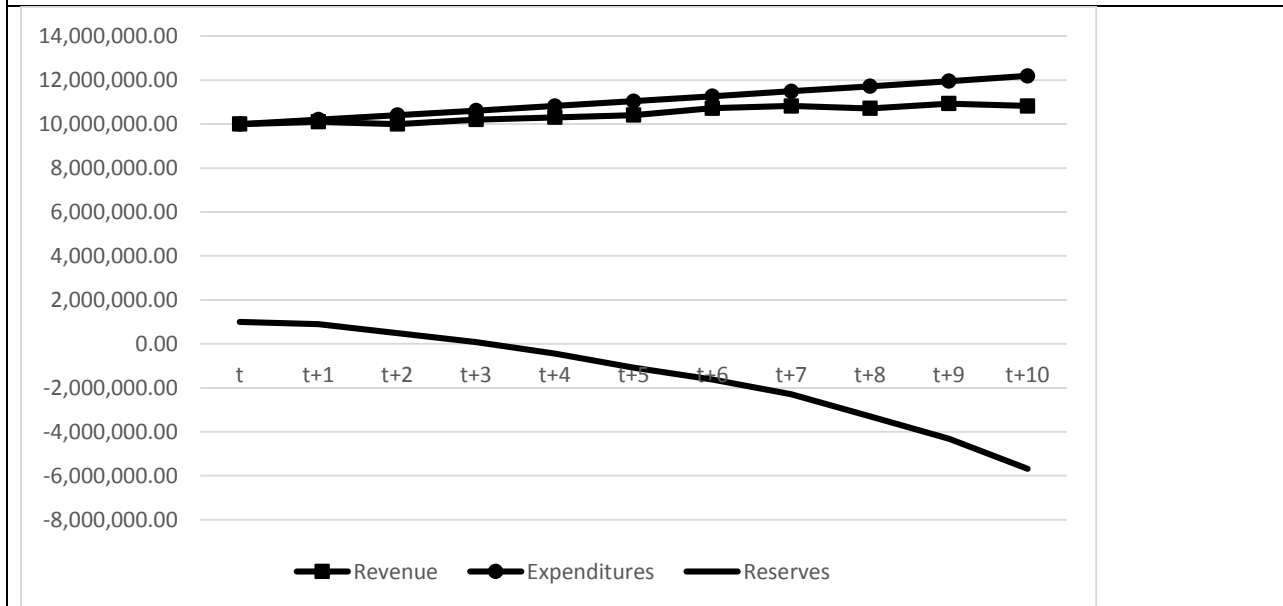
The effects of stochastic uncertainty can be shown in Figure 2. Panel A and Panel B both show outcomes 10 years into the future for a revenue process where the growth rate of revenue varies annually according to a uniform distribution with a minimum of -1 percent and maximum of 5 percent. Expenditure growth is a known value of 2 percent per year. In Panel A, the random outcome is that the initial level of reserves (10 percent of annual revenues) is sufficient to support revenues over the 10 year period. In Panel B, the random outcome is that the same initial level of reserves is nowhere near

sufficient to stabilize revenues. The key to understanding the effect of this uncertainty is that each of the outcomes is equally probable. Obviously the role of stochastic uncertainty greatly affects the evaluation of any level of fiscal reserves.

Figure 2. Effect of Stochastic Uncertainty on Reserve Levels.



Panel A.



Panel B.

Compounding this problem is the asymmetric nature of uncertainty. As many authors have noted, positive forecasting errors (underestimating future revenues or overestimating future expenditure demands) may have lesser consequences for governments and citizens than negative forecasting errors. This asymmetry may create a subtle bias toward forecasting a greater need for reserves than would be forecast if the consequences for forecasting errors were symmetric.

Another potential for structural breaks are breaks in the fiscal composition of governments. These can come from the decisions of a jurisdiction (for example, the state of Kansas drastically cutting reliance on the income tax in favor of sales taxes). They can also come from decisions of other jurisdictions that cause fiscal spillover effects (changes in federal Medicaid eligibility or compensation rules cause fiscal effects for state and local governments). These changes may raise or lower required reserves in ways that are not always easily forecastable.

Finally, implementing a plan for accumulating fiscal reserves may not be simple. Beyond the obvious political and programmatic pressures to spend more than any plan may mandate, there is a need to be able to distinguish between a real need for the expenditure of reserve resources and a manufactured need. There are obvious pressures to spend down reserves during even the mildest of economic downturns. This creates a need for firm and specific “release rules” that must be followed by governments.

Existing Academic Literature

While the literature on the determinants and use of fiscal reserves is somewhat more developed, the literature on the optimal size of those funds has received far less attention, possibly due to the problems identified above.

Table 1 lists each of the major pieces of literature in this topic area along with their focus on the risks facing state and local governments, the geographic nexus of data underlying their model, the model that they use to determine the optimal level of fiscal reserves and their findings.

Table 1. Existing Literature on Optimal Fiscal Reserves.

Author(s)	Source of Risk	Geography	Model	Findings
Vasche and Williams (1987)	Revenue forecasting errors	State of California	Confidence intervals	State should keep a minimum of 10% reserves
Navin and Navin (1997)	Economic and revenue risk	State of Ohio	Correlation/regression	Optimal fund balance of 13.51% with 68% confidence
Joyce (2001)	Economic, revenue, and expenditure (Medicaid)	Numerous states	Index construction/comparison	No "one size fits all" recommendation
Kriz (2003, 2002)	Revenue risk, desired expenditure growth	Minnesota local governments	Monte Carlo simulation	Required reserves 91.94% for average local government with 75% confidence
Dothan and Thompson (2006)	Economic (inflation and population growth), revenue, time	Hypothetical municipal government	Optimal control theory	Optimal reserve fund balance 93.57%

	preference for expenditures			
Marlowe (2011)	Fall in bond rating caused by inability to maintain spending, tax rates	Cities nationwide	Calculated change in probability of a given rating using ordered probit model	Reserves have minimal effect on rating probabilities

Vasche and Williams (1987) were the first to take up the question of how large fiscal reserves should be. They pointed out that this fundamental question was often left to the realm of politics and public opinion. They established that the need to use fiscal reserves to offset budgetary shortfalls had to be balanced against public opinion which often seems to feel that fiscal reserves are a “slush fund” that allows for currying political favors. They examined revenue forecasting errors from the state of California in order to project the amount of budget reserves that should be carried as a defensive budget mechanism. They conclude that the state should keep a minimum of 10 percent in fiscal reserves to cover potential shortfalls in the state budget.

Navin and Navin (1997) attempt to calculate the optimal size of and contribution rate to the Ohio Budget Stabilization Fund. They use a combination of regression analysis and simple trend analysis on state personal income and general fund own-source revenue. Analyzing the personal income data, they find that the average contribution period for building up reserves should be approximately 1-1/2 times as long as the average period when the fund would have to be available in order to stabilize spending. Examining the revenue data, they calculate that the optimal fund balance would be 13.51% during the period when the state was accumulating reserves. This would cushion the average revenue shortfall during a downturn with 68 percent confidence. If the state wished to be more confident in covering potential revenue shortfalls, they would need higher fund balances.

Joyce (2001) was one of the first to attempt to analyze “optimal” fiscal reserves across multiple jurisdictions. He developed an index of fiscal volatility through evaluating a state government’s reliance on the corporate income tax, federal aid, and gambling revenues along with the relative budget share of state Medicaid expenditures and the volatility of the state’s economic environment (measured by the difference between the state’s average unemployment rate and the average national unemployment rate for the 1990-97 period). He then relates the volatility index to the rainy day fund maintained in each state using a rank order type of analysis. He finds that some states with low volatility still maintain high balances and some states with high volatility maintain a low balance. He concludes that a “one size fits all” optimal rainy day fund balance prescription may not make sense.

Kriz (2003, 2002) pursued a more general model of determining fiscal reserve balances. His model was built up from the budget constraint of a jurisdiction. Implicitly, Kriz assumes a single budget decision maker who maximizes the utility of citizens in the jurisdiction through maintaining a certain level of expenditure growth over time. Kriz then introduces volatility into the model by specifying that the time path of revenues follows a Markov process (Geometric Brownian Motion). This introduces uncertainty as to whether the jurisdiction can fund the desired level of expenditures over time. Fiscal reserves are kept in order to smooth the revenue stream and allow the government to reach their desired expenditure growth level. Using data from larger Minnesota local governments over the period 1984-1999, Kriz solves for the level of fiscal reserves necessary to maintain various levels of expenditure growth. For a desired expenditure growth rate of 3 percent and rate of return on invested assets of 5 percent, if a government wanted to ensure that the expenditure growth rate would be maintained with 75 percent confidence, the required fiscal reserves would be 91.94 percent of annual revenues. For jurisdictions

with greater reliance on less volatile tax bases or with better returns on reserve investments, lower levels of reserves are required.

Dothan and Thompson (2006) do not analyze the topic of optimal fiscal reserves directly. Rather, they build a model of the optimal spending rate given several variables. Similar to Kriz, they build a model based on the budget constraint of a jurisdiction. Their model is built around a utility function of a budget policymaker that values a constant ratio of spending to wealth. Given an average growth rate and volatility of revenues, inflation and population growth, risk aversion coefficient, rate of time preference for expenditures, capitalization rate for expenditures, market price of risk, and rate of return on invested reserve fund assets, they solve for the optimal spending rate and reserve fund balance. Given a set of assumptions for a hypothetical jurisdiction, they find that the optimal spending rate of the jurisdiction should be approximately 2.4 percent of the wealth of the jurisdiction. The optimal reserve fund balance would be 93.57 percent of revenue. As the parameter assumptions are varied, the optimal spending rate does not change much. However, the optimal reserve fund balance varies dramatically, from a negative 200 percent of revenues for situations with high mean growth rates of revenues to 350 percent of revenues for situations with high inflation and population growth.

Marlowe (2011) takes a somewhat different approach to analyzing the optimal level of fiscal reserves. He relates the level of three measures of fiscal reserves (unreserved fund balance, total general fund balance, and unrestricted net assets in government funds) to bond ratings for a sample of over 500 cities. He derives predicted probabilities of credit ratings using an ordered probability model under different conditions of fiscal reserves, fiscal conditions, and demographic characteristics. He finds that slack resources (reserves) exhibit little effect on credit quality. Small, budget constrained jurisdictions

show the greatest reduction in the probability of obtaining a relatively low rating (A) if they keep a modest amount of slack versus a small amount. Larger, wealthy jurisdictions show a relatively strong increase in the probability of getting a prime (AAA) rating by keeping high levels of fiscal reserves (versus low levels). Otherwise the observed probability changes are mostly small and economically insignificant. He concludes that despite the rhetoric surrounding the maintenance of fiscal reserves, there is little evidence that keeping high levels of reserves reduces credit risk as measured by credit ratings.

Comparing Models

There are certainly many differences among the previous papers published on the topic of optimal fiscal reserves. The level of governments ranges from local to state, the geography varies from Ohio to California. There are three general or national level models (Joyce, Dothan/Thompson and Marlowe), while the others are geographic-specific, at least in their calibration. The modeling framework varies dramatically, from *ad hoc* examinations of what should be items that contribute to volatility (Joyce, e.g.) to fully defined systems based on the intertemporal budget constraint of a representative decision maker (Dothan and Thompson)

However, there are also many commonalities among the existing models. All of the models mention the need to maintain expenditures at a certain level as the reason for reserves. In the earlier models, this rationale was implicit. Kriz and Dothan/Thompson model explicitly the expenditure demands of jurisdictions. In Kriz, the model assumes that the expenditure demand is a choice of the budget decision maker, whereas in Dothan and Thompson the choice variable is the timing of expenditures. For Kriz, the “failure” of a given level of reserves is manifested in a need to reduce spending below the required level or increase tax rates to maintain the desired spending level. In Dothan and Thompson, failure would be

defined as an inability to maintain spending as a percentage of wealth as indicated by the model. All of the existing models rely heavily on revenue risk to motivate the level of need for budget reserves. In Vasche and Williams, this risk manifests itself through errors in revenue forecasts while in the other major models the risk is realized directly through fluctuations in revenue realizations.

There are also some common elements that are largely missing from the models. The first is that with the exception of Dothan and Thompson, there is no discussion of the costs of accumulating reserves. In Dothan and Thompson, there is a penalty for accumulating reserves in the form of reducing current spending and therefore utility of decision makers. But the other models only mention the cost of accumulating reserves in passing. The problem with the Dothan and Thompson paper, like the majority of the other papers (with the notable exception of Navin and Navin) is a definitive plan or path to accumulate reserves. In the Navin and Navin paper they estimate the period necessary to accumulate reserves from economic data, then use that and revenue volatility to calculate the required reserves.

Elements of a Synthesis Model

Developing a synthesis model for determining the optimal size of budget reserves is a task which has not heretofore been attempted in the academic literature. Each of the authors mentioned earlier appears to have been working very much independently. Still, we can use the information gleaned from reviewing those works as well as our basic knowledge of the role of budget reserves to sketch out the elements that should be present in a synthesis model of the optimal level of budget reserves. These are listed below with a short description:

- Economic uncertainty, either directly or indirectly through revenue uncertainty;

- Revenue uncertainty, in the form of:
 - Stochastic uncertainty about the future growth of the tax base and
 - Revenue forecasting errors;
- Expenditure demands, including the following dimensions:
 - Desired expenditure levels
 - Desired expenditure timing;
- Some recognition of the costs of accumulating reserves, either through displaced expenditure demand or a political “loss function”; and
- A plan for the accumulation of reserves, ensuring a long enough accumulation period to meet demands.

Forecasting-Simulation Model

In this section of the chapter, we develop one method for building a model of optimal budget reserves.

We term this method the “forecast-simulation” method because it involves building a forecast of key economic and financial variables for a jurisdiction, then simulating the economic/financial system into the future and assessing the likelihood of needing a reserve of a certain level. Our analysis involves the city of Omaha, Nebraska. Omaha was chosen due to the availability of a relatively long time-series of data on financial variables, which is necessary given our approach. Since the unit of analysis was not chosen randomly, this analysis should be viewed as a calibration of our approach.

We begin our approach by creating a time-series statistical model of the economy in the area that the jurisdiction encompasses, forecasting the economic variables into the future. Then we create a time-series statistical model of the financial variables in the jurisdiction that includes the economic variables as explanatory variables. Those financial variables should be affected by economic activity. We then generate forecasts of the key financial variables. Finally, we use the point estimates of the forecasts along with their associated standard errors to simulate the future financial situation of the city.

The variables used in the economic model are shown in Table 2 below, along with their sources. These variables are commonly used as indicators of an economy's health. As we have no intuition regarding the model specification, we use a vector autoregression (VAR) to forecast the variables. A VAR is a flexible model that uses the lagged values of one variable in a system of variables to predict the current values of itself and of the other variables in the system. The forecasting power comes from the fact that a VAR uses all of the information in the system of variables to predict the values of interest to forecasters versus using restrictions which variables affect other variables in the system. In vector notation, the system estimated was:

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \dots + \Pi_p Y_{t-p} + e_t, t = 1, 2, \dots, T \quad (1)$$

Table 2. Variables Used in the Forecasting-Simulation Model.

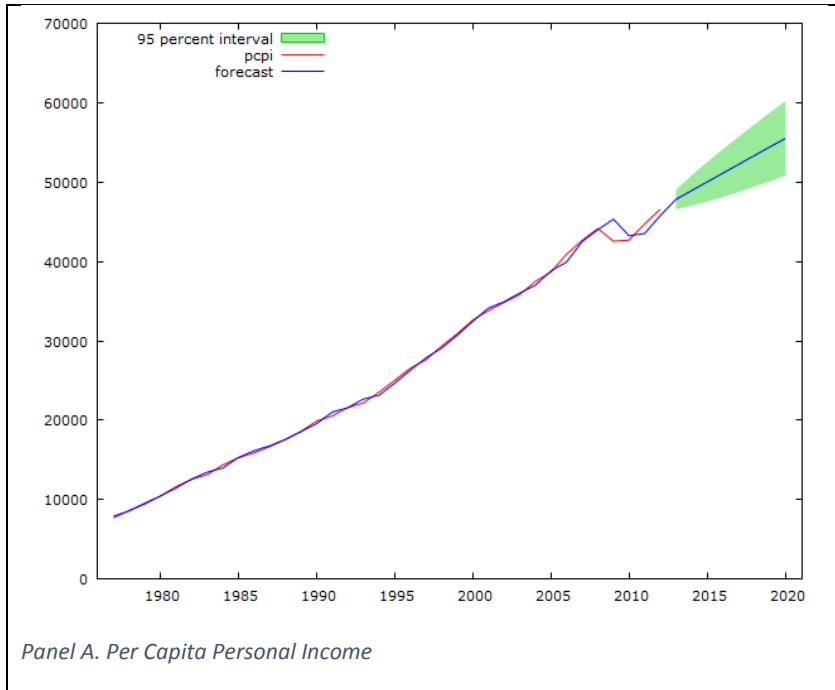
Variable	Source	Definition (Units)
PCPI	U.S. Bureau of Economic Analysis, Table CA 1-3	Per Capita Personal Income, Omaha-Council Bluffs MSA (dollars)
Wages	U.S. Bureau of Economic Analysis, Table CA 34	Wages and Salaries, Omaha-Council Bluffs MSA (thousands of dollars)
Emp	U.S. Bureau of Economic Analysis, Table CA 34	Wage and Salary Employment, Omaha-Council Bluffs MSA (jobs)
Taxable	Nebraska Department of Revenue, Monthly Taxable Retail Sales by City and County	Taxable Retail Sales in the City of Omaha, NE
Tot_Valuation	City of Omaha Annual Budgets, 1976 – 2015	Total Property Valuation
Sales and Use Tax	City of Omaha Annual Budgets, 1976 – 2015	General Fund Sales and Use Tax
Property Tax	City of Omaha Annual Budgets, 1976 – 2015	General Fund Property Tax and In Lieu of Taxes (PILOTs)
IGR	City of Omaha Annual Budgets, 1976 – 2015	General Fund Intergovernmental Revenue
Other_Revenues	City of Omaha Annual Budgets, 1976 – 2015	General Fund Revenue from the following categories: Municipal Enterprise Revenue (1975 – 1981), Business Taxes (1982 – 2013), Permits,

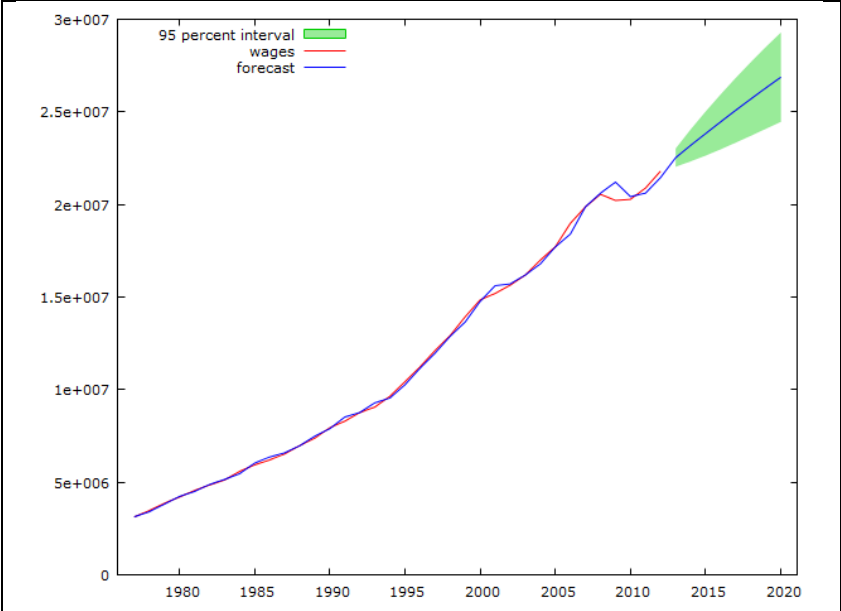
		Utility Occupation Taxes, and Cost Recovery Items, Miscellaneous Revenue, Restaurant Tax (2010 – 2013), Tobacco Tax (2013)
Expenditures	City of Omaha Annual Budgets, 1976 – 2015	General Fund Total Expenditures

The bold-faced Y vectors consist of the variables in the system, the Π matrices are coefficients relating a lag at time $t-p$ to the current value of the Y variables (Mills, 1990). The appropriate lag length, p , was chosen using the Schwarz-Bayesian Criterion and Portmanteau tests of serial correlation in the residuals of the system estimation.

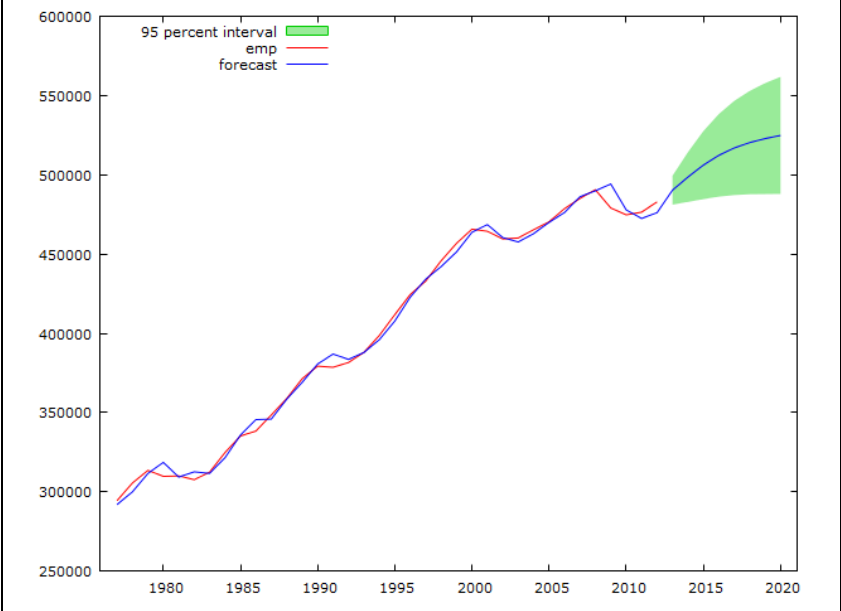
The results of the vector autoregression are shown in Appendix 1. A lag length of 2 was determined to be optimal. One note of caution about interpreting the results is that since the system is so heavily parameterized (it is only nearly identified) individual coefficient standard errors tend to be inflated, producing Type II biases in the interpretation of causal relationships. If our focus was hypothesis testing, we would typically present impulse response functions to interpret relationships. However, since our goal is to forecast the future values of the variables, we instead present graphs of the historical values of the variables (red line), the predicted and forecast values generated by the VAR (blue line) and the 95% confidence interval of the forecasts (green shaded area) in Figure 3. The model does a good job of predicting the values of the variables. Tests for serial correlation (using Ljung-Box p-values) autoregressive-conditional heteroscedasticity (ARCH), residual multivariate normality and cointegration all proved negative, indicating that the results are robust against the most important time-series threats to validity. The forecasts were then saved and entered into the next stage of the model.

Figure 3. Results of VAR Estimation of Equation (1), $T = 1977 - 2019$.





Panel B. Wages and Salaries



Panel C. Wage and Salary Employment

The financial forecast model was then run. Table 2 also details the variables included in the financial forecast model. For this analysis, we chose to focus on General Fund financial variables. Our model is

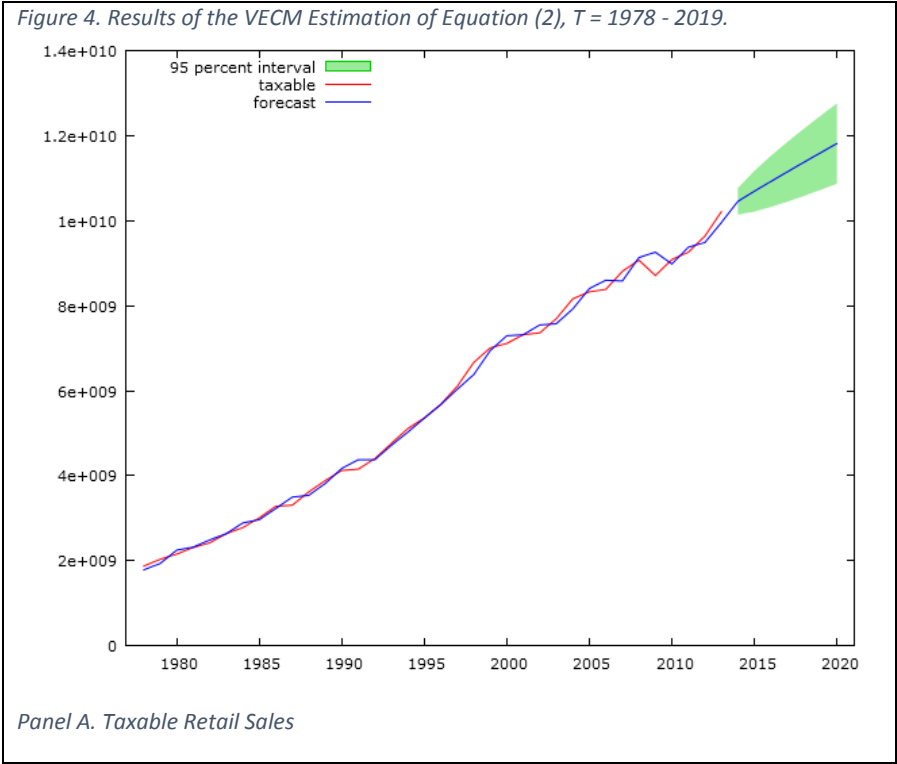
general, so it could be extended to include all Governmental Funds finances. Some of the revenue items were combined to form an Other Revenues variable. We model the base of the sales tax and property tax instead of their actual revenue realizations for two reasons. First, as we said earlier in the chapter, the volatility of those revenue sources are largely realized through volatility in the base. Second, to some extent property tax revenue and sales tax revenue (somewhat less so) are decided through policy terms. Later we will add a policy decision variable to the model to reflect that if policy changes are known, the jurisdiction may have to keep more or less reserves. We decided not to include Intergovernmental Revenue in the Other Revenues variable as it has a markedly different historical pattern than the other revenue sources. We also combined all of the expenditures into a Total Expenditures variable. When we first developed our model we had various categories of spending (e.g., Public Safety, General/Administrative) modeled separately. But the time series model lacked predictive power due to the relatively low degrees of freedom so we reduced the number of variables by combining expenditures.

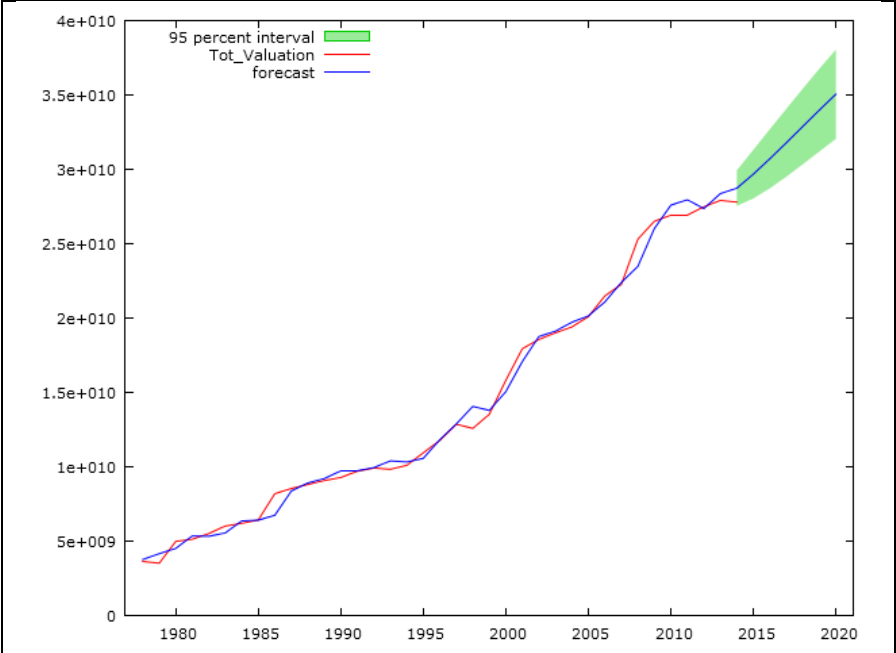
Initially we ran a VAR on the financial variables, including Taxable Retail Sales, Property Valuation, Intergovernmental Revenues, Other Revenues, and Total Expenditures as endogenous variables (Y variables in the VAR) and the forecasted economic variables from the first model estimation as exogenous variables (X variables). However, a cointegration test according to the Johansen method (Harris 1995) indicated the presence of cointegration – that the endogenous variables shared long-term trends. Therefore, we estimated a Vector Error-Correction model (VECM) to predict the variables:

$$\Delta Y_t = c + B_1 Y_{t-1} + \Pi_1 \Delta Y_{t-1} + \Pi_2 \Delta Y_{t-2} + e_t \quad (2)$$

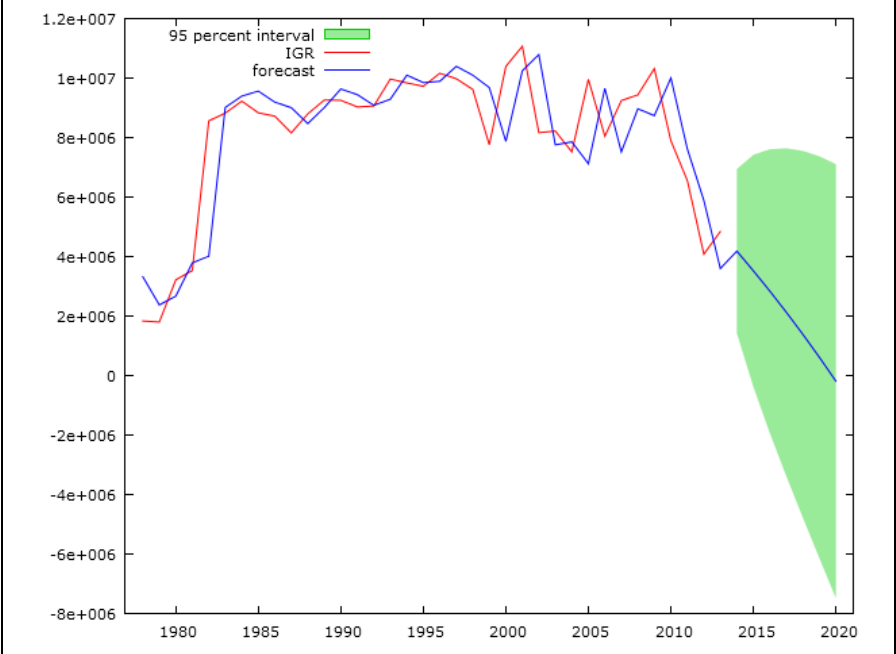
Here the Π matrices capture the short-run effects of changes in the endogenous variables on each other (the “error correction” matrices, while the B vector captures the long-run effects (the “cointegrating

vector”). Appendix 2 shows the statistical results of the VECM. Again, the tests of statistical significance are biased against rejection of the null hypothesis so interpretation is challenging. Tests of serial correlation, ARCH and multivariate normality for the residuals were negative, indicating robustness against threats to validity. Figure 4 shows the results of the forecast for each of the endogenous variables. The VECM produces predictions that fit the data very well for taxable retail sales, total property valuation, other revenues, and total expenditures. The model fits less well for intergovernmental revenues. The reason for the relatively poor fit lies in the inherent volatility of the revenue source, something noted earlier by Kriz (2003). There are clear upward trends in all of the other financial variables. But intergovernmental revenues resemble a “random walk” much more than having a clear trend. This volatility is reflected in the standard errors which are larger as a percentage of revenue generated. Once again, the forecasted values generated by the financial model were saved.

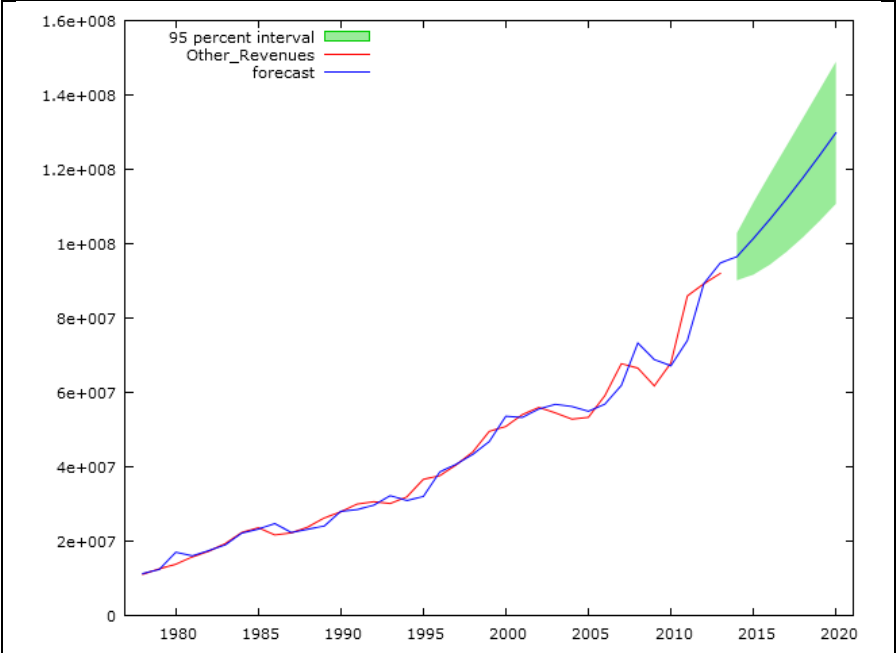




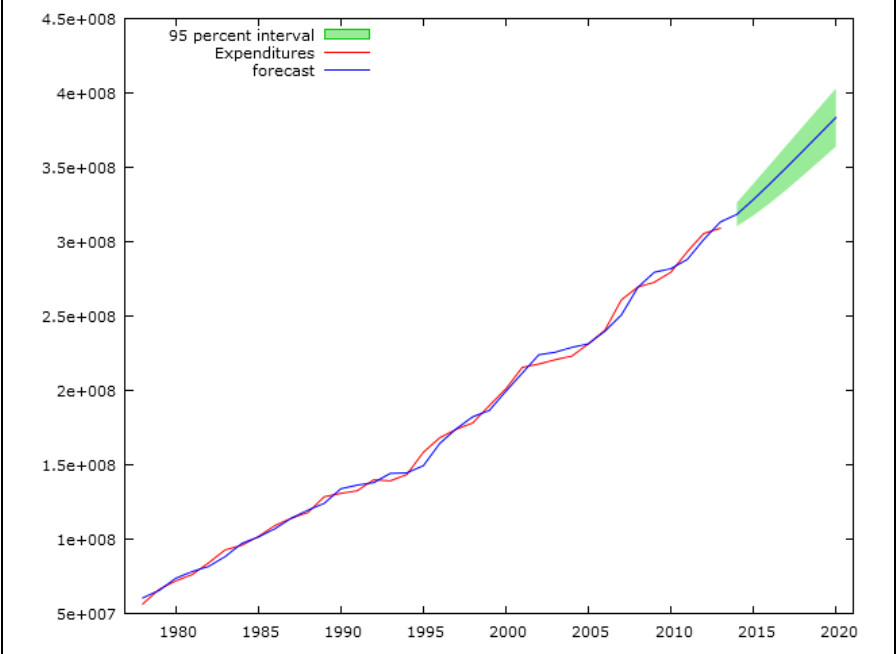
Panel B. Total Property Valuation



Panel C. Intergovernmental Revenues



Panel D. Other Revenues



Panel E. Total Expenditures

The final step of our model involved using the point estimates of the forecasts developed above along with their standard errors in a Monte Carlo simulation framework. In essence what we are doing is using the information generated by the forecast model to simulate future outcomes. In our model, we simulate only the current fiscal year (2014) and five years of future financial realizations. This keeps the model tractable and provides for recommendations that matter more to Mayors and City Councils that must make decisions about the level of reserves that affect cities currently and not far into the future.

Many authors have discussed the use of stochastic simulation models such as Monte Carlo simulation models in finance and public finance. As we said earlier, Kriz (2003) uses a similar model to determine the optimal level of budget reserves for cities in Minnesota. Our model differs from the earlier Kriz research because we are not using a naïve simulation. Kriz assumed a Markov process, whereas the forecasts generated by the models above contain information on the joint realization of financial variables, since they are estimated as a system using historical information. Therefore we can use the information generated by the model to simulate the future with less error than a naïve simulation approach.

Our simulation model is based on the following relationship:

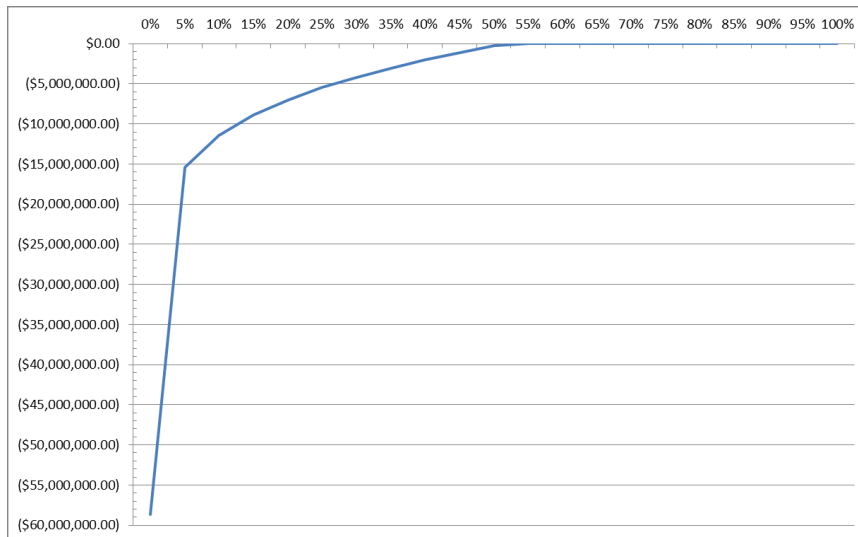
$$\widetilde{Balance}_t = \widetilde{Balance}_{t-1} + \widetilde{Taxable}_t * \widetilde{STYield} + \widetilde{Total\ Valuation}_t * \widetilde{PTYield} + \widetilde{IGR}_t + \widetilde{Other\ Revenues}_t - \widetilde{Total\ Expenditures}_t \quad (3)$$

The tilde over the various financial variables indicates an uncertain realization of the value of the variable. The uncertainty is determined by a random draw for the normal distribution with the parameters consisting of the point estimate from the forecast model for that year (mean) and standard deviation of that estimate. We also include a “yield” variable for determining the amount of sales taxes

and property taxes realized in each year. This yield can be thought of as the ratio of sales tax revenues to the level of taxable sales and the ratio of property tax revenues to total property valuation. We set the yield variables to the most recent value available (2013). In our model, each year that the balance goes below zero triggers a need for the use of reserves. We incorporate the need for a plan to accumulate reserves by discounting the need for reserves by 4 percent, our estimate of the cost of obtaining capital for a jurisdiction with a moderately strong credit rating. The resulting total of the present value of future deficits gives the optimal level of budget reserves that the jurisdiction should have now in order to cushion against future financial uncertainty.

Figure 5 shows the results of the simulation analysis. The horizontal axis shows the probability that the needed reserves will be at most that level. So for example, there is a 50 percent chance that the necessary reserves will be at most \$257,570. Therefore, if the city of Omaha wanted to be 50 percent certain that its reserves were sufficient to cover potential revenue shortfalls over the next 5 years, it should keep that amount of reserves on hand. If it wanted to be 75 percent certain it had adequate reserves (implying a 25 percent probability of a reserve realization), it should have \$5,478,917 in reserves. And if it wanted to be 95 percent certain, it should keep \$15,430,000 in reserves.

Figure 5. Results of the Monte Carlo Simulation of Equation (3), $T = 2014 - 2019$.



Conclusion

In this chapter, we have reviewed the existing literature on the determination of an optimal level of fiscal reserves. We have seen some of the challenges surrounding the determination of how large a fiscal reserve is necessary to stabilize revenues. We then compared the various models that have emerged in the literature and discussed their commonalities as well as unique features that each of them possess. We developed a framework for the creation of a model that synthesizes the best features from each of the models. Finally, we developed a model that addressed all of the features that should be present in a model of optimal budget reserves and discussed the results. Our model is scalable in that one could forecast necessary reserves with a longer period of analysis, using different costs of capital that may affect the need to accumulate reserves, or a change in the policies regarding property taxes and sales taxes (through different yield parameters). Therefore, it provides a strong alternative to existing models of optimal reserves. In conclusion, we feel that there is room for more research in this area, while at the same time admitting that developing a model that can achieve acceptance and be implemented will require a tough task.

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Appendix 1: Results of VAR Estimation of Equation (1)

VAR system, lag order 2
 OLS estimates, observations 1977-2012 (T = 36)
 Log-likelihood = -1085.1539
 Determinant of covariance matrix = 3.0520888e+022
 AIC = 61.4530
 BIC = 62.3767
 HQC = 61.7754
 Portmanteau test: LB(9) = 79.0089, df = 63 [0.0840]

Equation 1: pcpi
 Heteroskedasticity-robust standard errors, variant HC1

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	-431.502	1884.54	-0.2290	0.82050	
pcpi_1	0.905212	0.351493	2.5753	0.01538	**
pcpi_2	0.0462277	0.351894	0.1314	0.89639	
wages_1	0.000811061	0.000787011	1.0306	0.31127	
wages_2	-0.000765369	0.000899694	-0.8507	0.40191	
emp_1	-0.00324879	0.0198799	-0.1634	0.87132	
emp_2	0.00815657	0.0245793	0.3318	0.74239	
Mean dependent var	26003.11	S.D. dependent var	12129.07		
Sum squared resid	13083902	S.E. of regression	671.6912		
R-squared	0.997459	Adjusted R-squared	0.996933		
F(6, 29)	5851.423	P-value(F)	2.50e-43		
rho	0.077250	Durbin-Watson	1.800817		

F-tests of zero restrictions:

All lags of pcpi	F(2, 29) = 49.091 [0.0000]
All lags of wages	F(2, 29) = 0.83645 [0.4434]
All lags of emp	F(2, 29) = 0.17441 [0.8408]
All vars, lag 2	F(3, 29) = 0.75534 [0.5283]

Equation 2: wages
 Heteroskedasticity-robust standard errors, variant HC1

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	-363759	981913	-0.3705	0.71373	
pcpi_1	-0.986022	138.021	-0.0071	0.99435	
pcpi_2	34.4603	146.137	0.2358	0.81524	
wages_1	1.3742	0.308946	4.4480	0.00012	***
wages_2	-0.450504	0.34673	-1.2993	0.20408	
emp_1	5.03851	7.68838	0.6553	0.51741	

emp_2	-3.33156	8.77596	-0.3796	0.70699
Mean dependent var	11427235	S.D. dependent var	6030186	
Sum squared resid	2.09e+12	S.E. of regression	268396.5	
R-squared	0.998359	Adjusted R-squared	0.998019	
F(6, 29)	6184.220	P-value(F)	1.12e-43	
rho	0.153141	Durbin-Watson	1.653584	

F-tests of zero restrictions:

All lags of pcpi	F(2, 29) = 0.1702 [0.8443]
All lags of wages	F(2, 29) = 76.001 [0.0000]
All lags of emp	F(2, 29) = 0.29173 [0.7491]
All vars, lag 2	F(3, 29) = 4.5302 [0.0101]

Equation 3: emp

Heteroskedasticity-robust standard errors, variant HC1

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	35778.7	15863.4	2.2554	0.03182	**
pcpi_1	-2.81792	2.85557	-0.9868	0.33190	
pcpi_2	6.10315	2.72729	2.2378	0.03308	**
wages_1	0.00810603	0.00594357	1.3638	0.18311	
wages_2	-0.0134151	0.00624632	-2.1477	0.04023	**
emp_1	1.33965	0.164751	8.1314	<0.00001	***
emp_2	-0.483928	0.175906	-2.7511	0.01013	**

Mean dependent var	402204.2	S.D. dependent var	67236.55
Sum squared resid	7.10e+08	S.E. of regression	4947.875
R-squared	0.995513	Adjusted R-squared	0.994585
F(6, 29)	1230.833	P-value(F)	1.56e-33
rho	0.146150	Durbin-Watson	1.653598

F-tests of zero restrictions:

All lags of pcpi	F(2, 29) = 4.4234 [0.0211]
All lags of wages	F(2, 29) = 3.4668 [0.0447]
All lags of emp	F(2, 29) = 101.72 [0.0000]
All vars, lag 2	F(3, 29) = 11.869 [0.0000]

For the system as a whole

Null hypothesis: the longest lag is 1

Alternative hypothesis: the longest lag is 2

Likelihood ratio test: Chi-square(9) = 30.2676 [0.0004]

Appendix 2: Results of VAR Estimation of Equation (2)

VECM system, lag order 1
 Maximum likelihood estimates, observations 1978-2013 (T = 36)
 Cointegration rank = 1
 Case 3: Unrestricted constant
 beta (cointegrating vectors, standard errors in parentheses)

taxable 1.0000
 (0.00000)
 Tot_Valuation 4.4691
 (0.56250)
 IGR 473.11
 (184.39)
 Other_Revenues 816.16
 (123.13)
 Expenditures -872.29
 (101.62)

alpha (adjustment vectors)

taxable -0.022361
 Tot_Valuation -0.066538
 IGR -5.5796e-005
 Other_Revenues -0.00049628
 Expenditures 0.00028635

Log-likelihood = -3225.3701
 Determinant of covariance matrix = 4.5462475e+071
 AIC = 181.6872
 BIC = 183.6666
 HQC = 182.3781

Equation 1: d_taxable

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>
const	6.41861e+08	6.77783e+08	0.9470	0.35096
pcpi_hat	-5507.8	68022	-0.0810	0.93599
wageshat	11.0534	124.027	0.0891	0.92956
emp_hat	-3341.38	3143.78	-1.0629	0.29606
EC1	-0.0223609	0.0142185	-1.5727	0.12595
Mean dependent var	2.35e+08	S.D. dependent var		1.73e+08
Sum squared resid	9.11e+17	S.E. of regression		1.71e+08
R-squared	0.133900	Adjusted R-squared		0.022145
rho	-0.072995	Durbin-Watson		2.053530

Equation 2: d_Tot_Valuation

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	-1.74118e+09	2.58858e+09	-0.6726	0.50616	
pcpi_hat	-586640	259789	-2.2581	0.03112	**
wageshat	1002.97	473.683	2.1174	0.04235	**
emp_hat	8416.71	12006.7	0.7010	0.48853	
EC1	-0.0665383	0.0543032	-1.2253	0.22969	
Mean dependent var	6.82e+08	S.D. dependent var		7.21e+08	
Sum squared resid	1.33e+19	S.E. of regression		6.55e+08	
R-squared	0.269237	Adjusted R-squared		0.174945	
rho	0.126845	Durbin-Watson		1.733248	

Equation 3: d_IGR

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	2.01971e+06	5.9982e+06	0.3367	0.73860	
pcpi_hat	-274.587	601.977	-0.4561	0.65147	
wageshat	0.447271	1.09761	0.4075	0.68644	
emp_hat	-5.63573	27.8216	-0.2026	0.84080	
EC1	-5.57963e-05	0.00012583	-0.4434	0.66054	
Mean dependent var	56957.75	S.D. dependent var		1481306	
Sum squared resid	7.13e+13	S.E. of regression		1516794	
R-squared	0.071339	Adjusted R-squared		-0.048488	
rho	-0.208022	Durbin-Watson		2.354224	

Equation 4: d_Other_Revenue

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	1.91865e+07	1.38203e+07	1.3883	0.17495	
pcpi_hat	-2549.96	1387	-1.8385	0.07559	*
wageshat	5.16459	2.52897	2.0422	0.04972	**
emp_hat	-76.4452	64.103	-1.1925	0.24210	
EC1	-0.000496283	0.000289922	-1.7118	0.09693	*
Mean dependent var	2275671	S.D. dependent var		3670379	
Sum squared resid	3.79e+14	S.E. of regression		3494806	
R-squared	0.196995	Adjusted R-squared		0.093382	
rho	0.063778	Durbin-Watson		1.854034	

Equation 5: d_Expenditures

	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-ratio</i>	<i>p-value</i>	
const	-1.39068e+07	1.72304e+07	-0.8071	0.42575	
pcpi_hat	-1950.4	1729.23	-1.1279	0.26802	
wageshat	3.43215	3.15298	1.0885	0.28475	
emp_hat	111.187	79.9201	1.3912	0.17406	
EC1	0.000286347	0.000361459	0.7922	0.43427	

Mean dependent var	7156692	S.D. dependent var	4504985
Sum squared resid	5.89e+14	S.E. of regression	4357131
R-squared	0.171470	Adjusted R-squared	0.064563
rho	0.099787	Durbin-Watson	1.747921

Cross-equation covariance matrix:

	taxable	Tot_Valuation	IGR	Other_Revenues	Expenditures
taxable	2.5296e+016	-3.5145e+016	-2.6610e+013	8.1866e+013	9.1059e+013
Tot_Valuation	-3.5145e+016	3.6897e+017	2.0834e+014	-9.1711e+014	5.9302e+014
IGR	-2.6610e+013	2.0834e+014	1.9811e+012	-1.2952e+012	7.2969e+011
Other_Revenues	8.1866e+013	-9.1711e+014	-1.2952e+012	1.0517e+013	7.4125e+012
Expenditures	9.1059e+013	5.9302e+014	7.2969e+011	7.4125e+012	1.6348e+013

determinant = 4.54625e+071